Nondeterministic Turing Machines

Lecture 29
Section 10.3

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Outline

1. Nondeterminism
2. Examples
3. Assignment

Nondeterminism

- A nondeterministic Turing machine is defined like a standard Turing machine except for the transition function:

$$\delta : Q' \times \Gamma \to \mathcal{P}(Q \times \Gamma \times \{L, R\})$$

where $Q' = Q - \{q_{\text{accept}}, q_{\text{reject}}\}$.

Nondeterminism

- That is, $\delta(q, a)$ may result in any of a number of actions, including none.
- If *any* sequence of transitions leads to the accept state, then the input is accepted.
- If *all* sequences of transitions lead to the reject state, then the input is not accepted.
- Otherwise, the Turing Machine loops.

Nondeterminism

Theorem

Any language accepted by a nondeterministic Turing machine is also accepted by a standard Turing machine.

Nondeterminism

Proof.

- We may use a three-tape machine to simulate a nondeterministic Turing machine.
- Tape 1 preserves a copy of the original input.
- Tape 2 contains a “working” copy of the input.
- Tape 3 keeps track of the current state in the nondeterministic machine.

□

Nondeterminism

Proof.

- Start with the input w on Tape 1 and with Tapes 2 and 3 empty.
- Copy w from Tape 1 to Tape 2.
- For Tape 3, imagine the transitions starting from the start state as forming a tree.
- Each state has child states, namely, the states are that are accessible in one transition.

□

Nondeterminism

Proof.

- Let b be the largest number of children of any node.
- Number the children of each state using the numbers 1, 2, ..., b (as many as needed).

□

Nondeterminism

Proof.

- Now each finite string of numbers from $\{1, 2, \ldots, b\}$ represents a particular path through the nondeterministic Turing machine, or else represents no path at all.
- Begin by writing the empty string λ on Tape 3, representing no moves at all.

$\square$

Nondeterminism

Proof.

- If that leads to acceptance, then quit.
- If not, then replace λ with its lexicographical successor.
- For that string, follow the sequence of transitions that it describes.
- If that sequence leads to acceptance, then quit and accept.
- If not, then continue in the same manner.

$\square$

Nondeterminism

Proof.

- If w is accepted by the nondeterministic Turing machine, then some sequence of transitions leads to the accept state.
- Eventually that sequence will be written on Tape 3 and tried.
- If *all* sequences lead to the reject state, then at some point, every one of them will have been tried, resulting in rejection.
- Otherwise, there will be no path that has leads to acceptance and at least one path that will always avoid (keeping alive the false hope of acceptance) rejection, so the machine loops.

□

Outline

Examples

- How would we design nondeterministic Turing Machines to do the following?
  - Factor the input.
  - Find the “square root” of the input.
  - Recognize $\{w \mid n_{\mathbf{a}}(w) = n_{\mathbf{b}}(w)\}$.

Outline

Assignment

Homework

- Section 10.3 Exercises 2abd, 3.